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Mathematical Modelling and Experimental Study of the Cooling Rate of a Hot Fluid in Different Types of Containers

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ABSTRACT

Cooling of heated fluids is one of the critical aspects of the heat transfer and find extensive applications in engineering, science, and everyday life. Newton's Law of Cooling is the mathematical relation governing the cooling process where the heated body cools down upon being immersed into a cooler environment. The present study concerns itself with the mathematical modelling and experimental study of cooling rate of a hot fluid in different containers. In this experiment, hot water is the fluid used and it is placed in containers made from different types of material like ceramic, glass, and steel. This experiment was carried out in the humidity chamber so that the temperature and relative humidity of the surroundings could be kept constant. Fluid temperature was measured at various intervals of time and cooling curves were drawn for different types of containers to determine the cooling characteristics. Data obtained from experiments were evaluated graphically and using log functions to obtain values of cooling constant for various material containers used in the experiment. The graphs revealed that there is almost linear relationship in log form, hence supporting the nature of exponential cooling as predicted by the mathematical function. The cooling constant value (*k*) for each container was found to vary slightly based on the material used. This study shows the practical application of mathematical modeling to explain physical cooling process and highlights the significance of validation through experimentation for applied mathematics and thermal analysis.

Keywords: *Thermal Conductivity, Humidity Controlled, Differential Equations, Temperature, Thermal Analysis*

1 Introduction

Heat transfer is the process where heat flows naturally at the rate that it is transferred from a hot object to a cold object. Whenever a hot object is placed in a cold environment, heat will be lost until both are at the same temperature level, which is known as thermal equilibrium [4]. This transfer of heat takes place through three main modes, i.e. Conduction, Convection and Radiation. Understanding these three modes is very important for studying cooling processes and forms the basis of Newton's Law of Cooling, which used in this research [3]. Conduction is described as the process by which heat is transferred in a substance without any movement of that substance itself. Conduction mainly occurs in solids. When heat is applied to one part of a solid, atoms start vibrating rapidly to pass on their energy to the adjacent atoms [21]. The procedure keeps on going through the atoms until heat travels through the material. If you heat one end of a metallic rod, eventually, the other end will become warm. Conduction happens at different rates for different materials. Steel and metals have fast conductivity while ceramics are slow in this process. Mathematically, Using Fourier's law it can be expressed as:

$$q = -k\frac{dT}{dx}, \tag{1}$$

where *k* refers the thermal conductivity of material. The value of *k* is higher its means heat transfer process will be faster [10]. Convection refers to heat transfer taking place due to fluid motion. In this case, the hot fluid moves up while the cold fluid moves down, thus creating a current of fluids referred to as convection currents. When water is boiled in a pot, the hotter water tends to rise while the cooler water sinks for heating. This process helps in distributing heat throughout the fluid. Convection can be of two types: natural convection (due to temperature differences) and forced convection (due to external forces like fans). In cooling processes, convection is the most important mode because heat from the hot object is carries away by surrounding air. Mathematically, This process is represented by:

$$\frac{dT}{dt} = -k(T - T_a), \tag{2}$$

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which shows that cooling rate depends on difference between body and surroundings temperatures [1]. The process of heat transmission through electromagnetic waves is known as radiation. It does not need any medium to take place. Thus, heat can travel even when there is no medium between source and destination. The heat that we get from the Sun is transmitted to us through radiation. All bodies radiate heat and hot bodies radiate more heat than the cooler ones [26]. The loss of heat through radiation is calculated using the Stefan-Boltzmann Law:

$$Q = \sigma\epsilon A(T^4 - T_a^4), \tag{3}$$

where (σ) is a constant and (ϵ) depends on the surface of the object. Although radiation always occurs, its effect is usually smaller compared to conduction and convection in everyday cooling situations [17]. However, in real-life applications, all the above methods of heat transfer coexist. As an illustration, when cooling a hot fluid inside a container, the heat is initially conducted from the liquid to the walls of the container and then conveyed to the atmosphere by convection, with some loss of energy in the form of radiation. This process will depend on certain variables including the material of the container used, its surface area, temperature gradient, and ambient conditions such as air movement and moisture. From a mathematical point of view, heat transfer processes are classic examples of dynamic systems that can be described within the framework of differential equations. Among the earliest models for the process of heat transfer during cooling were those developed by Sir Isaac Newton in the early eighteenth century. According to the law of cooling enunciated by Newton, there is a relation between the change in temperature of an object with time and the temperature difference between the object and its surrounding medium. The process of cooling of hot fluids like water is quite common in our day-to-day life. Apart from that, in calculating the rate at which cooling takes place, there are some other physical factors apart from temperature that need to be taken into consideration. Some of them include the kind of material used for making the container in which the fluid is placed, the kind of contact surfaces it touches, and the nature of the fluid itself. Understanding this behavior is important in applications like food cooling, thermal management and industrial heat transfer systems.

2 Review of Literature

The discovery of Newton's Law of Cooling is known to be one of the very early and significant examples of applying mathematics in the field of natural science. This law was discovered by Sir Isaac Newton in 1701 and explains how the rate of cooling of a heated body depends on the temperature difference between the body and the surroundings Although quite simply formulated, it marked the beginning of the shift from purely qualitative analysis of nature to its quantitative modelling with mathematical tools [8]. In fact, by representing the temperature decrease of a heated object as its rate being proportional to temperature difference, Newton essentially created the basis of the differential equation as a tool for studying processes evolving in time [30]. Moreover, in addition to serving as a classical example of a differential equation, it represented the phenomenon of exponential decay. Interestingly enough, the equation remained unchanged and relevant even as scientists learned more about nature and physics in general and started distinguishing between three ways of heat transfer. During the development of thermodynamics and heat transfer theory in the 19th century [14]. The historical background and applications of Newton's Law of Cooling emphasize its importance in both theoretical and practical aspects of science. Since the development of the law by Newton and until now when the law is applied for practical purposes as well as theoretical modelling, it has been important to consider the basic ideas related to it and apply this knowledge effectively. Several researchers have studied cooling processes using theoretical and experimental approaches. Ferreira[9] investigated the impact of the surface exposure to heat and materials on cooling process. Wei et al.[28] managed to extend the use of cooling law to data migration processes in computing systems. Peker et al.[20] suggested new techniques for solving cooling equations analytically and presented their findings on new analytical techniques that help solve cooling equations efficiently. Jain[15] examined the situations in which law of cooling does not hold and the restrictions of this law under certain circumstances. Agayev et al.[1] used sensors to carry out cooling experiments accurately and found out new information about cooling process.Ambethkar and Basumatary [2] described the process of heat transfer in Newtonian fluid. Pandey et al.[19] examined the effect of coolant on the cooling process in engine oil by measuring the temperature with the help of an Arduino-based temperature sensor. It was realized that the presence of coolant results in faster cooling than natural cooling. This is important in showing how cooling systems play a role in cars and how useful it is to use Arduino in temperature sensing. Newton's Law of Cooling and its use for explaining how heat is lost from a body based on the temperature difference is explained by Winterton [29]. Arduino's application in physics experiments involving temperature dependency of resistance was illustrated by Sari and Kirindi [23], demonstrating its efficacy as an affordable and interactive educational device. Heat transfer with the help of electric fields and additive fluids is discussed by Saha et al.[22]. The use of Arduino in the design of the automatic temperature control system by Khaing et al. [16] shows how sensors and microcontrollers can be implemented to effectively monitor and regulate temperature. Measurement of convective heat transfer coefficient experimentally by

Conti et al.[7], allows one to quantify heat lost through body and surroundings, giving insight into convection heat transfer mechanisms. Silva et al.[25] utilize a smartphone for measuring cooling curves and recording data on heat loss in thermal experiments through the use of sensors available in smartphones. Schouten et al.[24] offer practical classroom experiments related to studying and minimizing evaporation and, thus, offer alternative means to study and analyze heat and mass transfer mechanisms. O'Schouten et al.[18] gives an insight into cooling law as well as the limitations of using it to describe the cooling of the object. Newton's Law of Cooling will be investigated experimentally using the Arduino tool provided by Galeriu [12], giving practical examples on how the data can be gathered to support the cooling phenomenon. The problematics surrounding uses Newton's Law of Cooling in teaching physics in today's environment are discussed by M.Vollmer[27], in this paper analyses Newton's Law in the light of history and relates it to other heat models.

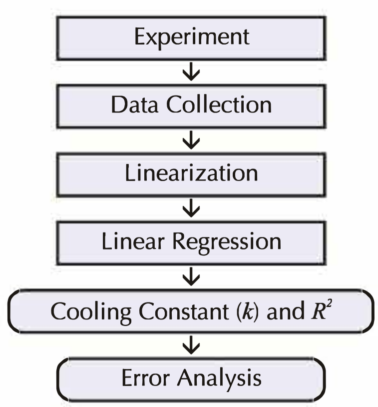
In the course of developing scientific knowledge, Cooling law was discovered not only as an abstract theory, but also as an empirical relation describing the dominating influences of convective heat transfer under experimental settings. Even though Newton's cooling law uses a constant proportionality constant, also called cooling constant, it was proved that this constant depended on other parameters related to physics such as the properties of the fluid around the body, the properties of the body itself, and the environment ([5], [21]). It was from this point on that further investigations led to more complicated models where the cooling constant became a function of time, thus creating nonlinearity in the model [11].

Although these improvements have been achieved, the Newtonian model is still largely applied due to its relatively simple analysis process and the ability to yield precise approximations under conditions with not too much variation in temperature [30].

Cooling law has a key application in modern sciences and engineering in a variety of different areas. The first major area of application is engineering where the law is commonly used for designing and analyzing cooling systems, including heat exchangers, thermal management in electronics, and industrial cooling, in which fast calculation of cooling is essential [6]. Environmental sciences use the law for modeling temperature change in natural systems, while biology and medicine use it for studying body temperature change and estimating time of death from a dead body's temperature [14].

3 Methods

Collection of data in this experiment started when all the set up parameters and environmental settings had been determined. The main reason why data was collected at this stage was that preliminary data of the variation of temperature with time in cooling of hot water using steel, glass and ceramic was needed. This was because of different thermal properties possessed by these substances. In this respect, each kind of material for the containers was examined three times. It is important to note that the equal amount of fluid at equal starting temperature was put into every tested container. The experiment was conducted at an ambient temperature of 20°C and a relative humidity of 50%. The container was placed in a humidity chamber where these conditions were maintained constant throughout the experiment. Temperature values were taken at specific time intervals until the fluid temperature reached the value of ambient temperature. The average of the three trials for each container was later used for analysis to minimize random experimental errors.



3.1 Materials and Apparatus

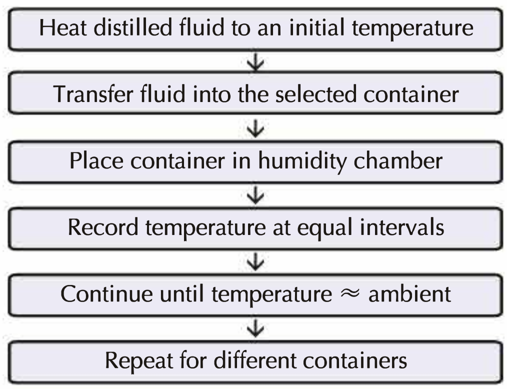
Following apparatus and materials were required for performing the experiment:

- Distilled water
- Symmetrical cylindrical containers made of different material including stainless steel, glass, and ceramic

- Calibrated laboratory thermometer
- Measuring cylinder to measure volumes
- Stopwatch or timer
- Humidity-controlled chamber
- Laboratory stand and holder
- Heating material like electric heating or gas burner

Before experimentation, each apparatus was examined to confirm its functionality and measurement precision. Hot water served as working fluid.

3.2 Experimental Process



3.2.1 Mathematical Modelling

The process of cooling the fluid can be explained using Cooling law, which states that the rate at which the temperature changes with respect to the body is directly proportional to the temperature difference between the body and its surroundings. Mathematically, Newton's Law of Cooling is given as:

$$\frac{dT}{dt} = -k(T - T_a), \tag{4}$$

where T refers to the temperature of the body at any time t, T_a refers to ambient temperature of the surrounding, and k refers to cooling constant.

This equation is a first order linear differential equation which will be the base of our mathematical model. Using the separation of variable, we will solve this equation, and the solution will be as follows:

$$T(t) = T_a + (T_0 - T_a)e^{-kt}, \tag{5}$$

where T_0 refers to the initial temperature of the body at $t = 0$.

This solution represents an exponential decay, which implies a rapid cooling process when the temperature difference is high and slow as equilibrium is achieved. This mathematical relationship has been validated through experiments in several research studies.

By taking the natural log of the equation above, we obtain the following linear relationship:

$$\ln(T - T_a) = \ln(T_0 - T_a) - kt. \tag{6}$$

This linear equation helps in finding the cooling constant through regression[13].

3.2.2 Data Collection and Analysis

Temperatures were recorded at intervals and presented in the form of tables for each container separately. Data obtained was studied in terms of their reliability and consistency. Verification of the mathematical model involved the presentation of data related to temperatures on a logarithmic scale. Graphs showing $ln(T - T_a)$ versus time were constructed. The regression equation was constructed using linear regression analysis.

Slope of the regression line equals the cooling constant multiplied by minus one ($-k$), while the coefficient of determination (R^2) determined its validity.

3.2.3 Error Minimization and Assumptions

For improving the accuracy of results attained through testing, the following was done:

- Conducting the experiment in controlled humidity conditions;
- Measuring the temperature at regular intervals;
- Conducting the experiment several times and averaging the data;
- Minimizing external factors like air currents.

Assumptions made included:

- Ambient temperature was constant;
- Fluid temperature was constant;
- Conformity of heat dissipation with the Cooling law.

4 Result and Discussion

Results obtained through this experiment are derived through observing the process of cooling in a heated liquid placed in ceramic, glass, and steel containers. Experimental data is gathered through temperature readings after particular intervals of time, and mathematical calculations are done using this information. Through these results, we can clearly see that our theory based on Cooling law works and prove the differences in cooling rates of the chosen materials.

4.1 Readings

The main data that have been recorded during experiments includes temperatures that have been recorded in various containers at fixed periods of time. The starting temperature of fluid was maintained approx the same for all three cases to ensure consistency and comparability of results. The ambient temperature was also recorded and kept nearly constant throughout the experiment. Table 1 represent the pilot data taken during the preliminary stage of experiment. The tables give an idea of the nature and consistency of the collected data. The temperature of the fluid was observed to decrease gradually with time in all three containers. During the initial stage of the experiment, the gap between the temperature of the liquid and the surrounding temperature was big and this showed by the sharp decline in temperature. With the passing of time, the temperature started falling less rapidly and gradually reached an asymptotic temperature level.

Although the overall trend was similar for all containers, noticeable differences were observed in the rate of cooling. The steel container showed a faster decrease in temperature compared to glass and ceramic, while the ceramic container retained heat for a longer duration. The behavior of the glass container was found to be intermediate.

4.2 Cooling Curves

In Figure 1 Cooling curves were plotted for each containers made of steel, glass, and ceramic, using the temperature-time data from Table 1 that shows how the temperature of a hot fluid decreases with time. In each case, the temperature is represented on the y-axis, and the time is represented on the x-axis. This makes the cooling process very easy to understand visually and can assist in verifying the Cooling law. The graph used to plot cooling is not linear but exponential in nature and decreasing. In the initial stages of cooling, when the hot fluid temperature is much greater than the surrounding, the rate of cooling is very high. But as the temperature difference reduces, the rate of cooling also reduces, and curve flattens out asymptotically towards ambient temperature.

The cooling curves indicate a steady fall in temperature over time. This indicates an exponential trend and confirms the Cooling law. There was differences in the slope of curves, which indicated that cooling rate depends on the material of container.It means the rate of cooling in a steel container was much faster than in glass, whereas comparatively a ceramic container has shown slow cooling behaviour.

Key Features of the Cooling Curves are:

Initial Steep Region

At the start, the curve is steep because temperature difference $(T - T_a)$ is maximum. According to Cooling law, the rate of cooling depends on the difference in temperature, and thus the initial cooling process will be faster.

Gradual Slope Decrease

The fluid will become cooler with time, meaning that the temperature difference becomes smaller, resulting in the slowing of the cooling process.

Final Flattening Region

At the latter stage, fluid temperature is closer to the surrounding temperature. It cools down very slowly, and the graph begins to flatten.

Table 1: *Average temperature changes of fluid over time in different types of containers*

Time (in min)	Ceramic (in °C)	Steel (in °C)	Glass (in °C)
0 (initial)	**90.0 (initial)**	**90.0 (initial)**	**90.0 (initial)**
2	81.0	79.0	81.0
4	72.5	70.0	72.0
6	66.5	63.0	65.0
8	61.0	58.0	59.0
10	57.5	53.5	56.0
12	54.5	49.5	53.0
14	51.5	46.0	50.0
16	49.0	44.0	48.0
18	47.0	42.0	46.0
20	45.0	40.5	44.5
22	43.0	38.0	43.0
24	41.5	37.0	41.0
26	40.0	36.0	39.0
28	38.5	35.0	37.0
30	37.0	33.5	36.0
32	35.5	32.0	35.0
34	34.0	31.5	33.5
36	33.0	31.0	32.0
38	32.5	30.0	31.5
40	32.0	29.5	31.0

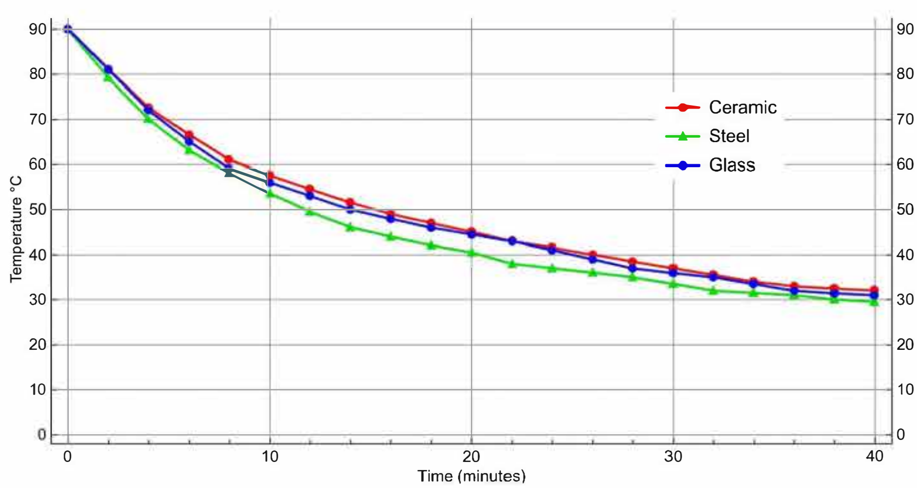


Fig. 1: *Cooling curves for ceramic, glass, and steel containers.*

Asymptotic Nature

The graph tends towards the ambient temperature line but does not touch it at any point of time in finite time interval. This is called asymptotic behavior.

Interpretation of Cooling Curves for Different types of Containers

The graphs for cooling rates with respect to the materials used for the containers (ceramic, glass, and steel) differ in the following way:

Steel Container: This graph is steep since it denotes rapid cooling due to high thermal conductivity.

Glass Container: The graph shows moderate steepness because of moderate cooling.

Ceramic Container: This graph is not so steep since it depicts slower cooling rates.

Thus, comparing cooling curves helps in understanding how material properties affect heat transfer.

4.3 Logarithmic Transformation of Data

The data for temperature and time is converted to a logarithmic scale and shown in Figure 2. The cooling process described in terms of the temperature versus time plot takes on an exponential nature, but one cannot easily find the cooling rate in the exponential cooling graph. This is why we use a logarithmic scale to obtain a linear relation.

Starting from the mathematical model:

$$T(t) = T_a + (T_0 - T_a)e^{-kt}, \tag{7}$$

we arrange it as:

$$T - T_a = (T_0 - T_a)e^{-kt}. \tag{8}$$

Taking natural logarithm on both sides:

$$ln(T - T_a) = ln(T_0 - T_a) - kt, \tag{9}$$

This equation represents a straight line of the form:

$$y = mx + c, \tag{10}$$

where $y = ln(T - T_a)$, $x = t$, slope $m = -k$ and intercept $c = (T_0 - T_a)$.

Therefore, the log graph plot is produced by plotting $\ln(T - T_a)$ along the vertical axis versus time t along the horizontal axis. In the case where the experimental data satisfies Cooling law, the plotted points follow a nearly straight line of negative slope. This shows that the relationship between the temperature and the time has a linear relationship.

However, in practical experiments, the plotted points do not lie exactly on a straight line due to small experimental errors such as fluctuations in ambient conditions, limitations of measuring instruments, and observational inaccuracies. In order to resolve this issue and obtain a more accurate approximation of the graphs trend, the best-fit line is drawn through

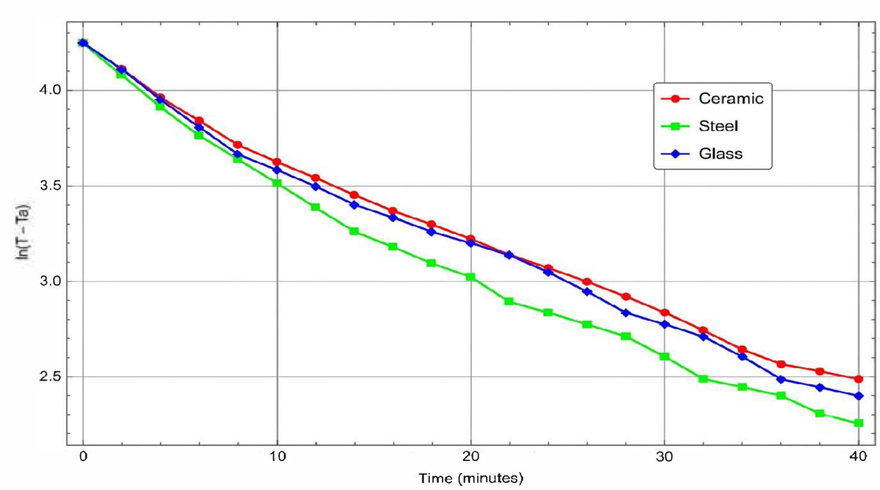


Fig. 2: *Log-linear plots of different types of contaniners*

the data points. Best fit line refers to a line that gives the overall description of the general trend of the data points, so that the total sum of all differences between the points and the line is the smallest possible. The simplest way of constructing the best fit line is the least squares approach that seeks to minimize the sum of the squares of errors or differences between the points and the line. The importance of using the best fit line is that we can estimate the value of k based on the slope of the graph as follows:

$$k = -(\text{slope of best fit line}). \tag{11}$$

The bigger the negative slope, the larger k, and the faster cooling we have, conversely if the negative slope is smaller, it means a slower rate of cooling. The y-intercept stands for $ln(T_0 - T_a)$. In this study, individual graphs of logarithms are generated for ceramic, glass, and steel containers. The graphs that have been fitted perfectly to the data sets for each type of container possess varying gradients because of the variation in their cooling rates. The container made of steel has the steepest gradient, followed by that of glass and finally, ceramic which has the least gradient. The combination of the logarithmic plot and best fit line makes a very good approach to the analysis of cooling. The reason for this is that while the log plot guarantees linear treatment of an exponential relationship, the best fit line helps in reducing the experimental errors associated with the experiment. In essence, this approach is critical to confirming the predictions made theoretically and determining the value of exact k.

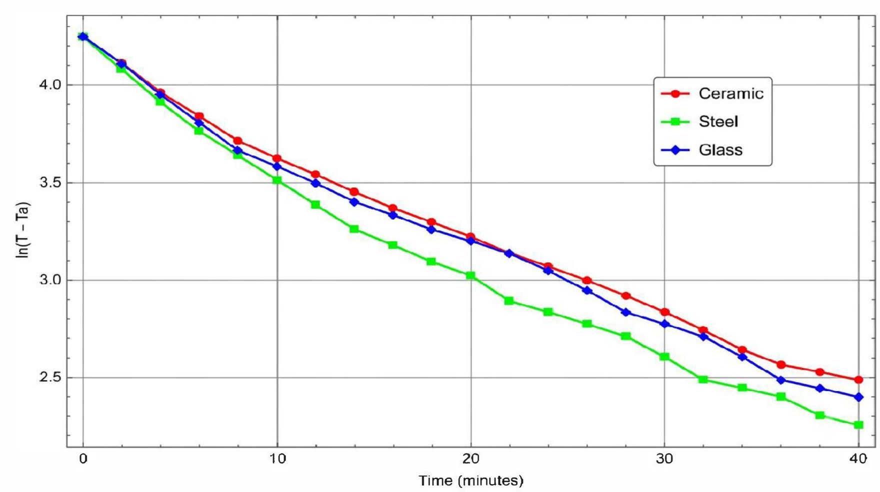


Fig. 3: *Best fit plot of different types of contaniners*

4.4 Comparative Analysis

Comparative study of cooling properties of various container materials is conducted by logarithmic graph $ln(T - T_a)$ versus time with the help of calculated cooling constant value. Such comparative study is based on the principles of Cooling law and implies a linear relationship between $ln(T - T_a)$ and time with k as the measure for comparison. From the experiment, the k of three materials were determined as:

Table 2: *Cooling Constant*

Container Material	Cooling Constant (k)	Container Material	Cooling Constant (k)	Container Material	Cooling Constant (k)
Ceramic	0.0427781	Steel	0.0479248	Glass	0.0442795

This means that the cooling rate depends greatly on the type of material from which the container is made. From the graph plotted on log scales, we observe that all the three data sets make nearly straight lines with negative slopes, thereby validating the application of Cooling law to our experiment. Nevertheless, lines have varying slopes because the cooling constants are not the same. From this graph, we notice that the value of $-k$ determines the steepness of the slope, which in turn influences the cooling process. Of the three containers, the steel container has the highest slope on the log graph and hence the highest cooling constant. The cooling rate in this case is the highest because heat is transferred from the liquid in the container more rapidly than in the other two containers. This is due to the high thermal conductivity of steel. The glass material exhibits an intermediate slope, thus having a cooling constant between those of the other materials considered. This gives glass cools slower as compared to steel but faster than ceramic. Glass has a thermal conductivity rate lower than that of steel but greater than ceramic. The container that is made of ceramic material has the least slope value, implying that it has the lowest cooling constant and hence the slowest cooling rate. Ceramics are good insulators, meaning that they limit heat transfer, and hence the fluid in them takes more time to lose heat.

The fact that the data points lie on the straight lines in the graph also indicates that the error in the experiment is low and the assumptions made for the mathematical model are quite valid. The deviations observed in the straight lines could possibly be due to environmental changes.

It can be clearly seen from the comparison that the fastest cooling occurs in steel, then in glass, and the slowest cooling occurs in ceramic. The determined series relates to the thermal conductivity of the material, which is theoretically correct. The selection of the logarithmic relationship and the best fit line not only prove that the cooling occurs exponentially but also assist in finding out cooling rate.

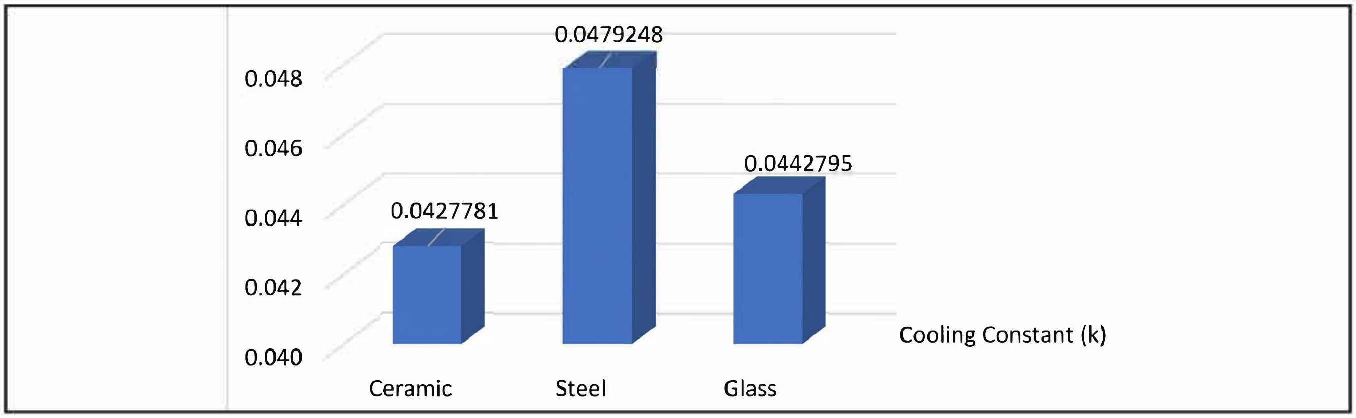


Fig. 4: *Cooling Constant for different types of contaniners*

4.5 Coefficient of Determination (R^2) and Error Calculation

In every experimental study that incorporates mathematical modeling, it becomes important not only to obtain the results but also to confirm their accuracy and reliability. The current study has used the principles laid out by Cooling law for describing the cooling of a heated fluid. This principle, once written mathematically and represented in the form of a logarithm, gives rise to a linear relationship between the variables $ln(T - T_a)$ and time. This enables us to apply certain statistical methods to verify the validity of our model as well as establish the accuracy of the results obtained experimentally. The most important of these methods include the coefficient of determination R^2 and error analysis.

The coefficient of determination, represented by R^2, is a mathematical statistic that assesses how accurately the regression or best-fit line reflects the actual observations. The coefficient of determination serves to mathematically quantify the percentage of variance between variables that can be attributed to one another. In layman's terms, the coefficient of determination reflects the accuracy of the experimental points relative to the theoretical straight-line model. Mathematically, it is defined as:

$$R^2 = 1 - \frac{\sum(y_i - y_{\text{fit}})^2}{\sum(y_i - \bar{y})^2}, \tag{12}$$

where y_i is the observed values, y_{fit} is the values predicted by the regression line and $\bar{y}$ is mean the observed values. The numerator in the above expression is the unexplained variance, while the denominator is the total variation. This means that R^2 basically tells you what percentage of the total variation has been explained by your model.

The R^2 value ranges from 0 to 1, and it is not complicated to understand. If $R^2 = 1$, it means the model fits perfectly, all the data points fall precisely on the regression line. On the other hand, a value close to zero indicates that the model does not adequately explain the data. In practical experimental situations, R^2 values above 0.95 are usually taken as an indication of a very good fit. For the three materials tested in this experiment, the R^2 values obtained are 0.989871 for the ceramic container, 0.978997 for the steel container, and 0.988545 for the glass container. The R^2 values obtained are very close to one, showing that there is a good linear correlation between the two parameters when put in log scale.

Table 3: *Coefficient of determination (R^2)*

Container Material	R^2 value	Container Material	R^2 value	Container Material	R^2 value
Ceramic	0.989871	Steel	0.978997	Glass	0.988545

A careful analysis of these values will show that R^2 of the ceramic container is higher than that of the glass container. It should also be noted that the steel container has the lowest R^2. Even though this difference is small, it can be justified by the characteristics of the material in question. Steel has higher thermal conductivity. As a result, the changes in the temperature happen so quickly that even tiny deviations from the ideal measurements can lead to errors, and, thus, higher deviation from the linear regression curve. On the other hand, ceramics do not conduct heat well. Therefore, the substance inside cools slowly, leading to better readings. The same trend applies to glass. However, the thermal conductivity of glass is intermediate, and thus its cooling speed lies somewhere in between the two materials mentioned before. Nevertheless, all of the three materials have R^2 values that prove how well the theory matches the data.

While the coefficient of determination examine the goodness of fit, it does not provide information about the accuracy of the estimated parameters, such as the cooling constant k. For this purpose, error analysis is carried out. In any experimental setup, it is impossible to eliminate all sources of error, and therefore it becomes necessary to quantify the uncertainty associated with the measured values. The k is obtained from slope the best fit line in the log plot which is subject to uncertainty due to various factors such as instrumental limitations, environmental conditions, and human observation.

The uncertainty in k is represented by Δk, which indicates the possible deviation of the measured value from the true value. To express this uncertainty in a meaningful way, the percentage error is calculated by:

$$\text{Percentage Error} = \left(\frac{\Delta k}{k}\right) \times 100. \tag{13}$$

This provides a relative measure of the accuracy of the experimental result in Table 4 and Table 5.

Table 4: *Uncertainty in the k (Δk)*

Container Material	(Δk)	Container Material	(Δk)	Container Material	(Δk)
Ceramic	0.000992757	Steel	0.00161039	Glass	0.0010935

Table 5: *Percentage error*

Container Material	Percentage Error	Container Material	Percentage Error	Container Material	Percentage Error
Ceramic	2.32071	Steel	3.36025	Glass	2.46955

In this study, the percentage errors for the three materials are small, ranging from around 2.3% to 3.6%. Small percentage errors show that the experiment was performed accurately, and also that the method adopted to determine the cooling constant is accurate. As seen before, the higher percentage error recorded for steel is due to the quick cooling of steel, making it difficult to record temperatures in the exact intervals. Ceramic takes time to cool down, hence recording of temperatures becomes more accurate, leading to smaller percentage errors.

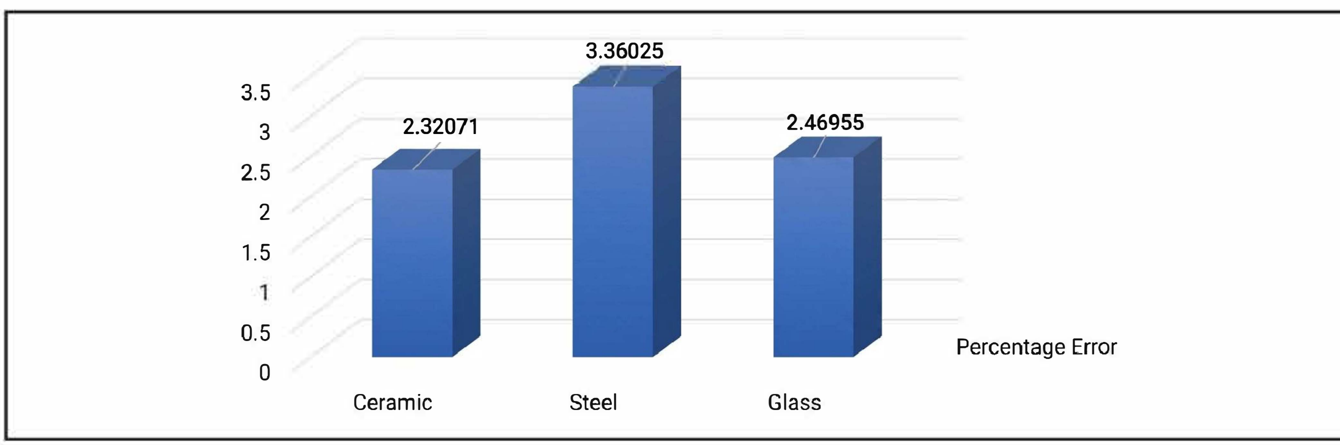


Fig. 5: *Percentage error for different types of contaniners*

Error sources in the experiment can be categorized into instrumental, observational, and environmental types. Instrumental error sources include errors that originate from the lack of accuracy of the thermometer employed in determining temperature. If the thermometer least count is 1 °C, all the readings will have uncertainties of ±0.5°C. Human error sources include errors that are caused by human incapability, such as delay in observing readings of the thermometer and time. Environmental error sources include errors that may arise due to changes in external conditions like ambient temperature, airflow, and moisture levels. Such environmental factors may influence the process of cooling and hence lead to deviations from the expected results.

In spite of all the above possible sources of error, the general reliability of this experiment is quite high, which can be seen from the percentage errors being very small and high R^2 values. The process of using logarithmic transformation along with fitting a best fit line will reduce the influence of random errors and thus make a better estimation of the cooling constant. Moreover, conducting the experiment under controlled conditions, where the ambient temperature is kept constant and other disturbances are minimal, means that the difference in observation comes from the material properties.

The analysis of the coefficient of determination and percentage error gives an overall assessment of the experiment. High values of R^2 show that the data is indeed following a linear trend after applying the logarithm operation to it. As a result, a relatively small percentage error shows that the experimentally determined constants of cooling are accurate enough. In conclusion, our study demonstrated high agreement between the theoretical prediction and experiment results, which shows the effectiveness of the chosen mathematical model.

5 Conclusion

Purpose of this study is to examine the cooling of a heated substance and prove the Newton's law of cooling through both calculations and practical experiments. Experiment involved observing the relationship between the rate of cooling and time for heated liquids in containers made of substances such as ceramic, glass, and metal. The results were analyzed using graphs and statistical tools. It became very evident through the experiments that the temperature of the liquid decreases constantly with respect to time, following an exponential trend. The phenomenon is completely in line with the mathematical theory which states that the rate of cooling depends upon the difference of temperature between the liquid and its surroundings.

The graphs obtained with respect to the relationship between temperature and time followed the conventional trend where temperature decreases rapidly initially and then slowly approaches the temperature of the surroundings. The logarithmic analysis of the data that was gathered became significant in confirming the theoretical model. Graphs plotted between natural log of (T - T_a) versus time produced graphs with linear characteristics on all three containers tested. This confirmed that the rate of cooling is indeed exponential. The gradients of the graphs produced the cooling constants. It should also be seen from the above results that the highest cooling constant was achieved for the steel container and the second highest was achieved for the glass container, whereas the lowest cooling constant was achieved for the ceramic container. This means that the maximum heat loss was obtained from the steel container, whereas the least heat loss was obtained from the ceramic container. This happens due to high thermal conductivity of steel and poor heat conductivity of ceramics. The comparative analysis clearly demonstrates the influence of material properties on the rate of heat transfer. The validity and reliability of the data that were acquired during the experiment were confirmed by the high values of the coefficient of determination (R^2), which was almost equal to 1 in all cases. This means that the experimental data were highly consistent with the theoretical model.

Furthermore, there were small errors in the calculation of the cooling constant that indicates the precision of the experimental process. Although there were some minor variations observed in the results, these could be attributed to various limitations in conducting the experiment, including measurement inaccuracies, changes in the environment, and human response time when recording data. However, these errors were within acceptable levels and had minimal impact on the findings. Finally, the experiment verifies the applicability and accuracy of Cooling law and showing its real world applications. It shows that there is an observable correlation between mathematical principles and physical observations, thus, emphasizing the significance of differential equations in describing heat transfer processes. It further proves the significance of material property in cooling process. Indeed, this is relevant in many fields including engineering and thermal management among others. Overall, this experiment shows that there is much truth in using mathematical models in predicting physical events in the real world.

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Conflict of Interest Statement

The authors declare that there is no conflict of interest regarding the publication of this paper.

Ethical Declarations

There was no involvement of human subjects or animals in this study, and there are no conflict of interest exists.

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